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إسألني
الهندسة
2013

تقدمان

دفتر

Calculus II

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ريم بركات

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* Rules

① $\int a \, dx = ax + c$

② $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \quad , \quad n \neq -1$

③ $\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{(n+1)(a)} + c \quad , \quad n \neq -1$

④ $\int x^{-1} \, dx = \int \frac{1}{x} \, dx = \ln|x| + c$

⑤ $\int \frac{1}{ax+b} \, dx = \frac{\ln|ax+b|}{a} + c$

⑥ $\int \cos(ax) \, dx = \frac{\sin(ax)}{a} + c$

⑦ $\int \sin(ax) \, dx = \frac{-\cos(ax)}{a} + c$

⑧ $\int \sec^2 x \, . dx = \tan x + c$

⑨ $\int \csc^2 x \, . dx = -\cot x + c$

(10) $\int \sec x \cdot \tan x \cdot dx = \sec x + c$

(11) $\int \csc x \cdot \cot x \; dx = -\csc x + c$

(12) $\int e^{ax+b} \, dx = \frac{e^{ax+b}}{a} + c$

(13) $\int \frac{1}{a^2 + x^2} \cdot dx = \frac{\tan^{-1}\left(\frac{x}{a}\right)}{a} + c$

(14) $\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + c$

(13) (15) $\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + c$

* (16) $\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

(17) $\int \sinh x \cdot dx = \cosh x + c$

(18) $\int \cosh x \cdot dx = \sinh x + c$

following ...

pf :- $\boxed{16}$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx$$

$$= \int \frac{1}{\sqrt{a^2\left(1 - \frac{x^2}{a^2}\right)}} dx = \int \frac{1}{a\sqrt{1 - \left(\frac{x}{a}\right)^2}} dx$$

let $\frac{x}{a} = z$ by substitution

لم نشتق بالنسبة x نشتق بالنسبة z

$$\frac{1}{a} dx = 1 \, dz \Longrightarrow \int \frac{1}{\sqrt{1 - z^2}} dz = \sin^{-1} z$$

$$= \sin^{-1}\left(\frac{x}{a}\right) + c \, .$$

*Ex :- Evaluate :-

① $\int \frac{1}{x} + \frac{1}{5 - 2x} + \sin(3x) + \frac{1}{4 + x^2} + 5 \, . dx$

$$= \ln|x| + \frac{\ln|5 - 2x|}{-2} - \frac{\cos(3x)}{3} + \frac{\tan^{-1}\left(\frac{x}{2}\right)}{2} + 5x + c$$

② $\int \frac{1}{\sqrt{5-x^2}} \cdot dx =$

$a^2 = 5$

$a = \sqrt{5}$

$\sin^{-1}\left(\frac{x}{\sqrt{5}}\right) + c$

③ $\int \tan x \cdot dx = \int \frac{\sin x}{\cos x} dx$

يلاني !!!

$z = \cos x$

$dz = -\sin x \, dx$

$-1 \, dz = \sin x \, dx$

$-\int \frac{1}{z} \cdot dz = -\ln|z|$

$= -\ln|\cos x| = \ln|\cos x|^{-1}$

$= \ln|\sec x|$

④ $\int \cot x \cdot dx = \ln|\sin x| + c$

⑤ $\int \sec x \, dx$

$z = \sec x + \tan x$

$dz = \sec x \tan x + \sec^2 x$

نضرب بالمرافق اللي
sec + tan

$= \int \sec x = \frac{\sec x}{1}\left(\frac{\sec x + \tan x}{\sec x + \tan x}\right)$

$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} = \ln|\sec x + \tan x| + c$

حفظ

18/2

-5-

For you H.w , $\int \csc x \, dx$

$-\ln|\csc x + \cot x|$

⑥ $\int \sin^2 x \, dx$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$= \frac{1}{2}(1-\cos 2x)$

$\frac{1}{2}\int 1-\cos 2x \, dx$

$= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + c$

⑦ $\int \cos^2 x \, dx$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$= \frac{1}{2}(1+\cos 2x)$

$\frac{1}{2}\int 1+\cos 2x \, dx$

$= \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + c$

7.1 Integration by parts [التكامل بالأجزاء]

$$\int \underset{\substack{\downarrow \\ \text{يسهل} \\ \text{اشتقاقه}}}{u} \cdot \underset{\substack{\downarrow \\ \text{يسهل} \\ \text{تكامله}}}{dv} = u \cdot v - \int v \cdot du$$

اللي بعرف أكامله بحطه dV

اللي بعرف أشتقه بحطه V

Use it usually with...

$\int \ln x \, dx$, $\int x^n \ln x \, dx$, $\int x^n e^x$, $\int x^n \underset{\cos x}{\sin x} \, dx$

Five Apple

6 –

* following ...

$\int$ inverse dx -------- $\int e^x \sin x \, dx$
$\cos x$

$\sin^{-1} x$

$\tan^{-1} x$

$\ln \rightarrow u$
inverse $\rightarrow u$

$\boxed{1}$ $\int \underset{u}{x^2} \cdot \underset{dv}{e^x} \, dx$

$= x^2 \cdot e^x - \int \underset{dv}{e^x} \cdot \underset{u}{2x} \, dx$

$= x^2 \cdot e^x - (2x \, e^x - \int e^x \cdot 2 \cdot dx)$

$= x^2 \cdot e^x - 2x \, e^x - 2 e^x + c$

* طريقة أخرى [طريقة الصفوفا]

اشتقاق		تكامل
x^2	+	e^x
$2x$	−	e^x
2	+	e^x
0		e^x

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. ملاحظة مهمة :

* Another way :-

** طريقة المصفوفات لا تصلح الا اذا كان اللي dv بدي تكامله exponetial واللي بشتقه (u) يا poly. يا sin يا cos.

$$= x^2 e^x - 2xe^x + 2e^x + C$$

Q. for you :- $\int x^3 e^{-x} \, dx$

$\int x^2 . \cos x \, . \, dx$

$\boxed{2}$ $\int \underset{u}{x} . \underset{dv}{\sin x} . dx$

$$= x(-\cos x) - \int (-\cos x) . 1 \, dx$$

$$+ \int \cos x . dx$$

$$= -x \cos x + \sin x + C .$$

$\boxed{3}$ $\int \underset{u}{\tan^{-1} x} . \underset{dv}{dx}$

$z = 1 + x^2$

$1 dz = 2x$

$\frac{1}{2} dz = x . dx$

$$= x \tan^{-1} x - \int x \frac{1}{1+x^2} dx$$

$$= x \tan^{-1} x - \frac{1}{2} \int \frac{1}{z} . dz$$

$$= x \tan^{-1} x - \frac{1}{2} \ln |z| = x \tan^{-1} x - \frac{1}{2} \ln |1+x^2| + C .$$

[4] $\int \underset{u}{e^x} \cdot \underset{dv}{\sin x}\, dx$

مابيفرق معك أي واحد u
وأي واحد dv

$$\int e^x \sin x \, dx = -e^x \cos x + \int \underset{u}{e^x} \cdot \underset{dv}{\cos x} \, dx$$

$$\int e^x \cdot \sin x \, dx = -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$$

$$2\int e^x \sin x \, dx = -e^x \cos x + e^x \sin x$$

$$\int e^x \sin x \, dx = \frac{-e^x \cos x + e^x \sin x}{2} + C$$

[5] $\int \underset{u}{\ln(2x+1)} \, \underset{dv}{dx}$

$$= x \ln(2x+1) - \int x \cdot \frac{2}{2x+1}$$

continue

$\boxed{z = 2x+1}$

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$$\int \frac{2x}{2x+1}\,dx = \int \frac{2x+1-1}{2x+1}\,.\,dx = \int 1 - \frac{1}{2x+1}$$

$$= x - \frac{\ln|2x+1|}{2}$$

[b] $\int \underbrace{x}_{dv} \cdot \underbrace{\ln x}_{u}\,dx$

$$= \frac{x^2}{2} \cdot \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x}\,.\,dx$$

$$\frac{1}{2}\int x\,.\,dx$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\left(\frac{x^2}{2}\right) + C$$

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** 7.2 Trigometric Integrals :-

** Power of sin :-

① $\int \sin x \, dx = -\cos x + c$

② $\int \sin^2 x \, dx = \frac{1}{2} \int 1 - \cos 2x \, dx$

$= \frac{1}{2} \left(x - \frac{\sin 2x}{2} \right) + C$

③ $\int \sin^3 x \, dx = \int \sin^2 x \cdot \sin x \, dx$

$\sin^2 x + \cos^2 x = 1$

$= \int (1 - \cos^2 x) \sin x \, dx$

$= -\int (1 - z^2) \, dz$

$z = \cos x$

$= -\left(z - \frac{z^3}{3} \right) + \cdots$

$dz = -\sin x \, dx$

⋮

(4) $\int \sin^5 x \, . \, dx$

$\int \sin^4 x \, . \, \sin x \, . dx$

$\int (\sin^2 x)^2 \, . \, \sin x \, . \, dx$

$= \int (1-\cos^2 x)^2 \, . \, \sin x \, . \, dx$

$Z = \cos x$

$1 dz = -\sin x \, . \, dx$

$= -\int (1-z^2)^2 \, . \, dz$

$= -\int 1 - 2z^2 + z^4 \, . \, dz$

$-\left(z + \frac{2z^3}{3} + \cancel{\frac{z^3}{3}} + \frac{z^5}{5} + c\right.$

بستخدم الاقترانات هاي

$\frac{1}{2}(1 \pm \cos 2x)$ التربيعي

$\sin^2 x + \cos^2 x = 1$ التكميلي

لما القوة فردية نفصل ①

زوجية ② "

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⑤ $\int \sin^4 x \, . \, dx$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\cos^2 2x = \frac{1+\cos 4x}{2}$

$= \int (\sin^2 x)^2 \, . \, dx$ $\qquad \frac{1}{2} + \frac{\cos 4x}{2}$

$= \int \left(\frac{1-\cos 2x}{2}\right)^2 . \, dx = \int \frac{1 - 2\cos 2x + \cos^2 4x \, . \, dx}{4}$

$= \frac{1}{4} \int 1 - 2\cos 2x + \frac{1}{2} + \frac{\cos 4x}{2} \, dx$

$= \frac{1}{4} \left(x - \frac{2 \sin 2x}{2} + \frac{1}{2} x + \frac{\sin 4x}{(2)(4)} + c \right.$

وال cos نفس الشي

⑥ $\int \cos^3 x \, . \, dx$

$= \int \cos^2 x \, . \, \cos x \, . \, dx$

$= \int (1 - \sin^2 x) . \cos x \, . \, dx$

$\sin x = z$

$\cos x \, . \, dx = 1 \, dz$

$\Longrightarrow \int (1 - z^2) \, . \, dz$

$= z - \frac{z^3}{3} + c$

$= \sin x - \frac{\sin^3 x}{3} + c$

3 حل آخر لرقم

$$\int \frac{\sin^3}{\cos^3} = \int \frac{\sin^2 x}{\cos^3 x} \cdot \sin x = \int \frac{1-\cos^2 x}{\cos^3 x} \cdot \sin x$$

$$z = \cos x \longrightarrow -\int \frac{1-z^2}{z^3} \cdot dz = -\int z^{-3} - \frac{1}{z}\, dz \ldots$$

* Powers of tan: ☺

$$\boxed{1} \int \tan x \; dx = -\ln|\cos x| = \ln|\sec x| + c$$

$$\boxed{2} \int \tan^2 x \cdot dx = \int \sec^2 x - 1 \cdot dx \qquad 1+\tan^2 x = \sec^2 x$$

$$= \tan x - x + c$$

$$\boxed{3} \int \tan^3 x \; dx = \int \tan x \cdot \tan^2 x \cdot dx$$

$$= \int \tan x\,(\sec^2 x - 1)\; dx$$

$$= \int \tan x \sec^2 x \cdot dx - \int \tan x \cdot dx$$

$$z = \tan x \qquad = \int z \cdot dz - \ln|\sec x|$$

$$dz = \sec^2 x \cdot dx$$

$$= \frac{z^2}{2} - \ln|\sec x|$$

$$= \frac{(\tan x)^2}{2} - \ln|\sec x| + c$$

④ $\int \tan^4 x \, . \, dx$

$= \int \tan^2 x \, . \, \tan^2 x \, . \, dx$

$= \int \tan^2 x \; (\sec^2 x - 1) \, . \, dx$

$= \int \tan^2 x \, . \, \sec^2 x \, . \, dx - \int \tan^2 x \, . \, dx$

$= \quad z = \tan x \qquad\qquad - \int \sec^2 x - 1$

$\quad dz = \sec^2 x \, . \, dx$

Done

$= \int z^2 \, . \, dz$ ---

⑤ $\int \tan^5 x \, . \, dx$ $\qquad \tan^4 x - \frac{3}{4} \tan^4 x - \frac{\tan^2 x}{2} + \ln |\sec x|$

$= \int \tan^3 x \, . \, \tan^2 x \, . \, dx$

$= \int \tan^3 x \, (\sec^2 x - 1) \, . \, d$

$= \int \tan^3 x \, . \, \sec^2 x \, . \, dx - \int \tan^3 \, . \, dx$

$z = \tan x \qquad \int z^3 \, . \, dz$ --- ؟؟ هيك الو تكملة

Same as cos

أنا قوة في ما بنجبر

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* Power of sec :-

① $\int \sec x \, dx = \ln|\sec x + \tan x| + c$

② $\int \sec^2 x \, dx = \tan x + c$

③ $\int \sec^3 x \, dx$

$= \int \underbrace{\sec^2 x}_{dv} \cdot \underbrace{\sec x}_{u} \, dx$ (by parts)

$= \int \sec^3 x \, dx = \sec x \cdot \tan x - \int \tan x \cdot \sec x \cdot \tan x \, dx$

$\tan^2 x = \sec^2 x - 1$

$= \int \sec^3 x \, dx = \sec x \cdot \tan x - \int \sec x (\sec^2 x - 1) \, dx$

$= \int \sec^3 x \, dx = \sec x \cdot \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$

$= 2 \int \sec^3 x \, dx = \sec x \cdot \tan x + \ln|\sec x \cdot \tan x|$

$\Rightarrow \int \sec^3 x \, dx = \dfrac{\sec x \cdot \tan x + \ln|\sec x + \tan x|}{2} + c$

csc as sec

* ①: $\int \sec^4 x \, dx$

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① بال sin وبال cos بنفصل

2 بال tan بنفصل

Sun. 23/2/2014

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* Reduction Formula :-

لـبـس حفـظ

① $\int \sin^n . dx = -\frac{1}{n} \cos x . \sin x^{n-1} + \frac{n-1}{n} \int \sin x^{n-2} x . dx$, $n \geq 2$

② $\int \cos^n . dx = \frac{1}{n} \cos^{n-1} x . \sin x + \frac{n-1}{n} \int \cos^{n-2} x . dx$, $n \geq 2$

③ $\int \tan^n . dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x . dx$, $n \geq 2$

* pf ① :-

$$\int \sin^n x . dx = \int \underbrace{\sin^{n-1} x}_{u} . \underbrace{\sin x . dx}_{dv} \quad \text{(By Parts)}$$

$$\int \sin^n x . dx = \sin^{n-1} x . (-\cos x) - \int (-\cos x) . (n-1) \sin^{(n-2)} x . \cos x . dx$$

$\cos^2 x = 1 - \sin^2 x$

$$\int \sin^n x . dx = -\sin^{n-1} x . \cos x + (n-1) \int (1 - \sin^2 x) \sin^{n-2} x . dx$$

$$\int \sin^n x . dx = -\sin^{n-1} x . \cos x + (n-1) \int \sin^{(n-2)} x . dx - (n-1) \int \sin^n x . dx$$

$\boxed{n-1+1 = n}$

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$$\int \sin^n x \cdot dx = -\sin^{n-1} x \cdot \cos x + (n-1)\int \sin^{n-2} x \cdot dx - (n-1)\int \sin^n x \cdot dx$$

$$n\int \sin^n x \cdot dx = -\sin^{n-1} x \cdot \cos x + (n-1)\int \sin^{n-2} x \cdot dx$$

$$\int \sin^n x \cdot dx = \frac{-1}{n} \sin^{n-1} x \cdot \cos x + \frac{(n-1)}{n}\int \sin^{n-2} x \cdot dx$$

* Powers of sin and cos :- $\int \sin^n x \cdot \cos^m x \cdot dx$

$\boxed{1}$ If the power of sin is odd. (وما حكى اشي عن ال cos)

Ex:- $\int \sin^3 x \cdot \cos^2 x \cdot dx$

$\int \sin^2 x \cdot \cos^2 x \cdot \sin x \cdot dx$ → مشتقة cos

$\int (1-\cos^2 x) \cdot \cos^2 x \cdot \sin x \cdot dx$

$$\left\{ \begin{array}{l} \cos x = z \\ -\sin x \, dx = 1 dz \end{array} \right.$$

$-\int (1-z^2) \cdot z^2 \cdot dz = -\int z^2 - z^4 \cdot dx$ نكامل

* نفرض الزوجي "

2 If the power of cos is odd :-

Ex :- $\int \sin^4 x \cdot \cos^5 x \cdot dx$

$\int \sin^4 x \cdot \cos^4 x \cdot \cos x \cdot dx$ مشتقة sin

$\int \sin^4 x \cdot (1-\sin^2 x)^2 \cdot \cos x \cdot dx$

$z = \sin x$

$dz = \cos x \, dx$

$\int z^4 (1-z^2)^2 \cdot dz$

$= \int z^4 (1 - 2z^2 + z^4)$ ....

* نفك ثم نكامل

علة

3 If n and m are odd :-

Ex :- $\int \sin^3 x \cdot \cos^3 x \cdot dx$

** (نفصل وحدة منهم يا sin يا cos)

وما نفصلهم الاثنين

$\int \sin^2 \cdot \cos^3 x \cdot \sin x \cdot dx$ مشتقة cos

$\int (1-\cos^2 x) \cdot \cos^5 x \cdot \sin x \cdot dx$

$-\int (1-z^2) z^3 \cdot dz$

$z = \cos x$

$dz = -\sin x$

$\boxed{4}$ If n and m are even :-

Remember ^^

$\sin 2x = 2\sin x \cos x$

$\frac{\sin 2x}{2} = \sin x \cdot \cos x$

Ex:- $\int \sin^2 x \cdot \cos^2 x \cdot dx$

$= \int (\sin x \cdot \cos x)^2 \cdot dx = \int \left(\frac{\sin 2x}{2}\right)^2 \cdot dx$

$= \frac{1}{4}\int \sin^2 2x \cdot dx$

$= \frac{1}{4}\int \frac{1-\cos 4x}{2}\, dx = \frac{1}{8}\int 1-\cos 4x \cdot dx$

$= \frac{1}{8}\left(x - \frac{\sin 4x}{4}\right) + c$

والطريقة السابقة لا تصلح إلا لما يكونوا نفس الأس يعني $\begin{matrix}2 & 2\\ 4 & 4\end{matrix}$

↳ Another way using طريقة أخرى

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\sin^2 x \cdot \cos^2 x = \frac{(1-\cos 2x)}{2} \cdot \frac{(1+\cos 2x)}{2} = \frac{1-\cos^2 2x}{4}$

$= \frac{1}{4}(1-\cos^2 2x)$

$= \frac{1}{4}\left(1 - \frac{1}{2} - \frac{\cos 4x}{2}\right)$

$= \frac{1}{4}\left(\frac{1}{2} - \frac{\cos 4x}{2}\right)$

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following $\int \sin^2 x \cdot \cos^2 x \cdot dx = \frac{1}{4}\int \frac{1}{2} - \frac{\cos 4x}{2}$

$= \frac{1}{4}\left(\frac{1}{2}x - \frac{\sin 4x}{8} + c\right)$

* Powers of sec and tan :-

[1] if the power of sec is even

→ separate (نفصل) $\sec^2 x$ and use

$1 + \tan^2 x = \sec^2 x$. " نفرض ال tan u "

Ex :- $\int \sec^4 x \cdot \tan^2 x \cdot dx$

$\int \sec^2 x \cdot \tan^2 x \cdot \sec^2 x$

مشتقة ال tan

خلي الباقي بدلالة tan :

$\int (1 + \tan^2 x)\tan^2 x \cdot \sec^2 x \cdot dx$

$z = \tan x$

$1 dz = \sec^2 x \cdot dx$

$\int (1 + z^2) \cdot z^2 \cdot dz$

⋮

نوزع ثم نكامل

$\boxed{2}$ If the power of tan is odd

⟹ separate (نفصل) $\sec x \cdot \tan x$

Ex:- $\int \sec^3 x \cdot \tan^5 x \; dx$

$\int \sec^2 x \cdot \tan^4 x \cdot \underbrace{\sec x \cdot \tan x}_{} \cdot dx$ → sec مشتقة ال

$\int \sec^2 x \; (\sec^2 x - 1)^2 \cdot \sec x \cdot \tan x \; dx$

$\int z^2 \; (z^2 - 1)^2 \cdot dz$ || $z = \sec x$

⋮

Ex: $\int \sec^4 x \cdot \tan^3 x \cdot dx$

1st way :- $\int \sec^2 x \cdot \tan^3 x \cdot \sec^2 x \cdot dx$

$= \int (1 + \tan^2 x) \tan^3 x \cdot \sec^2 x \cdot dx$

- - - -

2nd way :- $\int \sec^3 x \cdot \tan^2 x \cdot \sec x \cdot \tan x \; dx$

$= \int \sec^3 x \; (\sec^2 x - 1) \; \underbrace{\sec x \cdot \tan x}_{\text{مشتقة } \sec} \; dx$

- - -

$U = \tan x$

$dx = \frac{dv}{\sec x}$

Ex:- $\int \sec x \cdot \tan^2 x$

$$\int \sec x \cdot (\sec^2 x - 1) \cdot dx$$

$$\int \sec^3 x \, . dx - \int \sec x . dx$$

By parts $- \ln |\sec x + \tan x| + c$

* Identities:-

حفظ

[1] $\sin A \cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$

[2] $\cos A \cos B = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$

[3] $\sin A \sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$

* Ex:-

$$\int \sin(\overset{A}{3x}) \cdot \cos(\overset{B}{4x}) \, dx$$

Sol:-

$$\int \frac{1}{2}\left((\sin 7x + \sin(-x)\right) dx$$

$$= \frac{1}{2}\left(\frac{-\cos 7x}{7} - \frac{\cos(-x)}{-1}\right) + C$$

Examples:-

[1] $\int e^{\sqrt{x}} \cdot dx$

let $\boxed{\sqrt{x} = z}$

$\frac{1}{2\sqrt{x}} = 1\, dz$

$\boxed{dx} = 2\sqrt{x} \cdot dz = 2z \cdot dz$

$\int \underbrace{e^{z}}_{dv} \cdot \underbrace{2z}_{u}$ ---- By parts :

$2\sqrt{x}\, e^{\sqrt{x}} - e^{\sqrt{x}} + c$

[2] $\int x^3 \cdot e^{x^2} \cdot dz$

$= \int x \cdot x^2 \cdot e^{x^2} \cdot dx$ || $z = x^2$, $1dz = 2x \cdot dx$

$= \frac{1}{2} \int \underbrace{z}_{u} \cdot \underbrace{e^{z}}_{dv}$

------ By parts :

[3] $\int \underbrace{\sin(\ln x)}_{u} \cdot \underbrace{dx}_{dv}$

$= \int \sin(\ln x) \cdot dx = x \sin(\ln x) - \int \cancel{x} \cdot \cos(\ln x) \cdot \frac{1}{\cancel{x}} \cdot dx$

$= \int \sin(\ln x) \cdot dx = x \cdot \sin(\ln x) - \int \underbrace{1}_{dv} \cdot \underbrace{\cos(\ln x)}_{u} \cdot dx$

Follow →

following

$$\int \sin(\ln x)\,.dx = x\,.\sin(\ln x) - (x\,.\cos(\ln x) - \int x\,.(-\sin \ln x)\,.\frac{1}{x}\,.dx$$

$$= \int \sin(\ln x)\,.dx = x\,\sin(\ln x) - x\,.\cos(\ln x) - \int \sin(\ln x)\,dx\,.$$

دوري

$$2\int \sin(\ln x)\,.dx = x\,.\sin(\ln x) - x\,.\cos(\ln x)$$

---- By parts.

ومنه نظري

$$\frac{x\sin(\ln x) - x\cos(\ln x)}{2}$$

طريقة اخرى

$$Z = \ln x \longrightarrow x = e^{z}$$

$$1\,dx = e^{z}\,.dz$$

$$\int e^{z}\,.\sin z\,.dz \quad ------$$

By part دوري

^ ^

* 7.3 Trigonometric Substitution :-

$\sqrt{a^2+b^2x^2}$, $\sqrt{a^2-b^2x^2}$, $\sqrt{b^2x^2-a^2}$

Let $bx = a\tan\theta$, $bx = a.\sin\theta$, $bx = a\sec\theta$

Ex :-

$$\int \frac{x}{\sqrt{9-x^2}} \cdot dx$$

$$= \frac{-1}{2} \int \frac{1}{\sqrt{z}} \cdot dz$$

$z = 9-x^2$

$dz = -2x \cdot dx$

$$= \frac{-1}{2} \int z^{-\frac{1}{2}} \cdot dz$$

$$= \frac{-\frac{1}{2}\, z^{\frac{1}{2}}}{\frac{1}{2}} + c$$

Ex:

$$\int \frac{x^2}{\sqrt{9-x^2}}\,dx$$

using Trig. sub.

let $x = 3\sin\theta$

$\boxed{dx = 3\cos\theta\,.\,d\theta}$

كل اشي بدلالة θ

$\boxed{x^2 = 9\sin^2\theta}$

$9 - x^2 = 9 - 9\sin^2\theta$

$= 9(1-\sin^2\theta)$

$= 9\cos^2\theta$

$\sqrt{9-x^2} = 3\cos\theta$

$$\int \frac{9\sin^2\theta}{3\cos\theta}\,.\,3\cos\theta = 9\int \sin^2\theta\,.\,d\theta$$

$$= \frac{9}{2}\int 1-\cos 2\theta\,.\,d\theta$$

$$= \frac{9}{2}\left(\theta - \frac{\sin 2\theta}{2}\right)$$

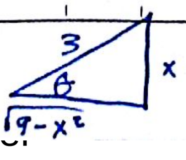
بعدين θ نرجع نعوضه بدلالة x

$$= \frac{9}{2}\left(\theta - \frac{\cancel{2}\sin\theta\cos\theta}{\cancel{2}}\right) = \frac{9}{2}\left(\theta - \sin\theta\,.\cos\theta\right)$$

$x = 3\sin\theta$

$\sin\theta = \frac{x}{3}$

$\theta = \sin^{-1}\frac{x}{3}$



$$\frac{9}{2}\left(\sin^{-1}\frac{x}{3} - \frac{x}{3}\,.\,\frac{\sqrt{9-x^2}}{3}\right) + C$$

Ex:-

$$\int e^x \cdot \sqrt{1-e^{2x}} \cdot dx$$

$$= \int e^x \cdot \sqrt{1-(e^x)^2} \cdot dx$$

$$e^x = z \longrightarrow e^x \cdot dx = 1\, dz$$

$$\int \sqrt{\underbrace{1}-\underbrace{1z^2}} \cdot dz$$

Trig. Sub. $\boxed{1z = 1\sin\theta}$

$$\boxed{1\, dz = \cos\theta \cdot d\theta}$$

$$1 - z^2 = 1 - \sin^2\theta = \cos^2\theta$$

$$\boxed{\sqrt{1-z^2} = \cos\theta}$$

$z \to \theta \to \theta$

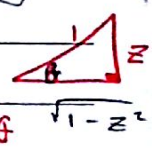
$$\int \cos\theta \cdot \cos\theta \cdot d\theta$$

$$= \int \cos^2\theta \cdot d\theta$$

$$= \frac{1}{2}\int 1 + \cos 2\theta \cdot d\theta$$

$$= \frac{1}{2}\left(\theta + \frac{\sin 2\theta}{2}\right)$$

$$= \frac{1}{2}\left(\theta + \frac{\cancel{2}\sin\theta \cdot \cos\theta}{\cancel{2}}\right)$$



$z = \sin\theta$

$\sin^{-1} z = \theta$

$$\frac{1}{2}\left(\theta + \sin\theta \cdot \cos\theta\right) = \frac{1}{2}\left(\sin^{-1} z + z\sqrt{1-z^2}\right)$$

$$\frac{1}{2}\left(\sin^{-1} e^x + e^x \cdot \sqrt{1-e^{2x}}\right) + C$$

27/2/2014

* $\sqrt{a^2+b^2x^2}$

$(1+x^2)^{\frac{3}{2}} = \left((1+x^2)^{\frac{1}{2}}\right)^3$

let $bx = a\tan\theta$

Ex:

$$\int \frac{dx}{(1+x^2)^{\frac{3}{2}}}$$

$$= \int \frac{dx}{(\sqrt{1+x^2})^3}$$

= Trig. sub. let $1x = 1\tan\theta$

$$\boxed{1\,dx = \sec^2\theta \, . \, d\theta}$$

$1+x^2 = \tan^2\theta + 1 = \sec^2\theta$

$\sqrt{1+x^2} = \sec\theta$

$\sqrt{1+x^2} = \sec\theta$

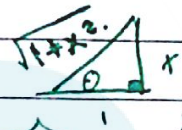
$(\sqrt{1+x^2})^3 = \sec^3\theta$

follow

$$\int \frac{\sec^2\theta \cdot d\theta}{\sec^3\theta} = \int \frac{1}{\sec\theta} \cdot d\theta$$

$$= \int \cos\theta \cdot d\theta = \sin\theta$$

$$\frac{x}{1} = \tan\theta$$



$$= \frac{x}{\sqrt{1+x^2}} + C$$

* $\sqrt{b^2x^2 - a^2}$

let $bx = a \cdot \sec\theta$.

Ex.:
$$\int \frac{dx}{x^2\sqrt{4x^2-9}}$$

Sol: Trig. Subs. let $2x = 3\sec\theta$

$x = \frac{3}{2}\sec\theta$

$$\boxed{dx = \frac{3}{2}\sec\theta \cdot \tan\theta \cdot d\theta}$$

$x^2 = \frac{9}{4}\sec^2\theta$

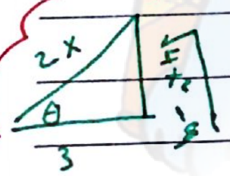
$$4x^2 - 9 = 9\sec^2\theta - 9$$
$$= 9(\sec^2\theta - 1)$$
$$= 9\tan^2\theta$$

30-

$$\boxed{\sqrt{4x^2-9} = 3\tan\theta}$$

$$\int \frac{\frac{3}{2}\sec\theta \cdot \tan\theta}{\frac{9}{4}\sec^2\theta \cdot 3\tan\theta} = \frac{2}{9}\int \frac{1}{\sec\theta} \cdot d\theta$$

$$= \frac{2}{9}\int \cos\theta \cdot d\theta = \frac{2}{9}\sin\theta$$



$$= \frac{2}{9} \cdot \frac{\sqrt{4x^2-9}}{2x} + C$$

$2x = 3\sec\theta$

$\frac{2x}{3} = \sec\theta$

31 –

* $\sqrt{ax^2+bx+c}$, $\sqrt{ax^2+bx}$

Complete the square , the continue

ما بيعني الـ constant

أول لما أشوف جذر وتحته دالة

عطول إكمال مربع

معامل

coeff. of $x^2 = 1$

$$\left(\frac{\text{coeff of } x}{2}\right)^2 = \# \quad +,-$$

Ex.:

$$\int \frac{dx}{\sqrt{2x-x^2}}$$

$$\left(\frac{-2}{2}\right)^2 = 1$$

$$2x - x^2 = -x^2 + 2x =$$

$$-(x^2-2x) = -(x^2-2x+1-1)$$

$$= -\left((x-1)^2 - 1\right)$$

$$= 1 - (x-1)^2$$

let

$x - 1 = z$

$1\,dx = dz$

$$\int \frac{dx}{\sqrt{1-(x-1)^2}}$$

$$= \int \frac{dz}{\sqrt{1-z^2}} = \sin^{-1} z$$

$$= \sin^{-1}(x-1) + C$$

السلام

3Q

$$\int \sqrt{6-(x-1)^2}\,.dx$$

Sol:-

let $x-1 = z \rightsquigarrow dx = dz$

$$\int \sqrt{6-1z^2}\,.\,dz$$

Trig. sub.

let $1z = \sqrt{6}\,\sin\theta$

$$\boxed{1\,dz = \sqrt{6}\,\cos\theta\,.\,d\theta}$$

$$6-z^2 = 6-6\sin^2\theta$$
$$= 6\,(1-\sin^2\theta) = 6\,.\cos^2\theta$$

$$\boxed{\sqrt{6-z^2} = \sqrt{6}\,\cos\theta}$$

$$\int \sqrt{6}\,.\cos\theta\,.\sqrt{6}\,.\,\cos\theta\,.\,d\theta$$

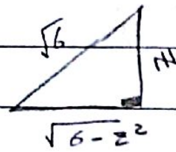
$$= 6\int \cos^2\theta\,.\,d\theta$$

$$= 3\int 1+\cos 2\theta\,.\,d\theta = 3\left(\theta + \frac{\sin 2\theta}{2}\right)$$

$$= 3\left(\theta + \frac{2\sin\theta\,.\,\cos\theta}{2}\right)$$

Five Apple

$$3\left(\sin^{-1}\frac{z}{\sqrt{6}}+\frac{z}{\sqrt{6}}\cdot\frac{\sqrt{6-z^2}}{\sqrt{6}}\right)+c$$



Ex:

$$\int \frac{x\,.\,dx}{x^2+6x+10}$$

completing seq.

$$x^2+6x+10 = x^2+6x+9-9+10$$

$$= (x+3)^2+1$$

$$\Rightarrow \int \frac{x\,.\,dx}{(x+3)^2+1}$$

let $x+3 = z$

$1\,dx = dz$

$x = z-3$

$$\int \frac{z-3}{z^2+1}\,.\,dz$$

وزعت البسط على المقام

$$\int \frac{z}{z^2+1}\,.\,dz - 3\int\frac{1}{z^2+1}\,.\,dz$$

tan

$$= \frac{1}{2}\ln\left|z^2+1\right| - 3\tan^{-1} z$$

$$= \frac{1}{2}\ln\left|(x+3)^2+1\right| - 3\tan^{-1}(x+3) + c$$

مباشرة لأن البسط هو مشتقة المقام

Another Way

$z = \tan\theta$ [Trig. sub.]

$1\,dz = \sec^2\theta \cdot d\theta$

$z^2+1 = \tan^2\theta + 1 = \sec^2\theta$

$$\int \frac{\tan\theta - 3}{\cancel{\sec^2\theta}} \cdot \cancel{\sec^2\theta} \cdot d\theta$$

$$\int \tan\theta - 3\,d\theta = -\ln|\cos\theta| - 3\theta$$

. - - -

- -- . Continue . . . -

2/3/2014

* Q:- $\displaystyle\int \frac{\cos x}{\sqrt{1+\sin^2 x}} \cdot dx$

let $z = \sin x \rightsquigarrow dz = \cos x \cdot dx$

$\displaystyle\int \frac{dz}{\sqrt{1+z^2}}$ Trig. sub. let $z = \tan\theta$.

* Q:- $\displaystyle\int \sqrt{x(4-x)} \cdot dx$

$= \displaystyle\int \sqrt{4x - x^2} \cdot dx$

completing square

---- ---

** 7.4 Integration by partial fraction :-

use when you have $\dfrac{\text{Poly.}}{\text{Poly.}} = \dfrac{P(x)}{Q(x)}$

→ if degree $P(x) <$ degree $Q(x)$.

* درجة المقام أكبر من درجة البسط *

II) If $Q(x) = (x-a)(x-b)(x-c) \ldots$ distinct linear factors. (دالة جذور مختلفة)

then $\frac{P(x)}{Q(x)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c} + \ldots$

$$\int \frac{P(x)}{Q(x)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c} + \ldots$$

$$= A \ln|x-a| + B \ln|x-b| + C \ln|x-c| + \ldots$$

So we need to find A, B, C

Ex:- $\int \frac{x}{x^2+5x+6}$

Sol:-

$x^2+5x+6 = (x+2)(x+3)$ distinct Linear

$$\frac{x}{x^2+5x+6} = \frac{A}{x+2} + \frac{B}{x+3} = \frac{A(x+3)+B(x+2)}{(x+3)(x+2)}$$

المقام = المقام ∴ البسط = البسط

$$\Rightarrow \boxed{x = A(x+3) + B(x+2)} \quad \forall x$$

شو اللي بصفر الأقواس

let $x = 3$ $\boxed{B=3}$

let $x = -2 \Rightarrow \boxed{A=-2}$

$$\int \frac{x}{x^2+5x+6}\,dx = \int \frac{-2}{x+2} + \frac{3}{x+3}\,.dx$$

$$= -2\ln|x+2| + 3\ln|x+3| + c$$

★ Another way to find A, B

$$x = A(x+3) + B(x+2)$$

$$x = Ax + 3A + Bx + 2B$$

$$x = (A+B)x + (3A+2B)$$

cofficiend of x are equal.

$$\Rightarrow \quad 1 = A+B$$

$$0 = 3A+2B$$

$$-2 = -2A - 2B$$

$$0 = 3A + 2B$$

$$-2 = A$$

$$1 = -2 + B$$

$$B = 3$$

Ex:

$$\int \frac{x^2+2x-1}{2x^3+3x^2-2x}\,dx$$

Sol:- $2x^3+3x^2-2x = x(2x^2+3x-2) = x(2x-1)(x+2)$

distinct
مختلف

$$\Rightarrow \frac{x^2+2x-1}{2x^3+3x^2-2x} = \frac{A}{x} + \frac{B}{2x-1} + \frac{C}{x+2}$$

$$\frac{x^2+2x-1}{2x^3+3x^2-2x} = \frac{A(2x-1)(x+2) + Bx(x+2) + Cx(2x-1)}{x(2x-1)(x+2)}$$

$$x^2+2x-1 = A(2x-1)(x+2) + Bx(x+2) + C.x(2x-1)$$

Let $x=0 \Rightarrow -1 = -2A \rightsquigarrow A = \frac{1}{2}$

Let $x=-2 \Rightarrow -1 = 10C \rightsquigarrow C = \frac{-1}{10}$

Let $x = \frac{1}{2} \Rightarrow \frac{1}{4} = \frac{5}{4}B \rightsquigarrow B = \frac{1}{5}$

$$\int \frac{\frac{1}{2}}{x} + \frac{\frac{1}{5}}{2x-1} + \frac{\frac{-1}{10}}{x+2}\,dx = \frac{1}{2}\ln|x| + \frac{1}{5}\,\frac{\ln|2x-1|}{2} + \frac{-1}{10}\ln|x+2| + c$$

$\boxed{2}$ $Q(x) = (x-r)(x-r)(x-r)\dots$ repeated roots متكررة

$$\Rightarrow \frac{P(x)}{G(x)} = \frac{A}{x-r} + \frac{B}{(x-r)^2} + \frac{C}{(x-r)^3} + \dots$$

* Ex: Write the partial decomposition for :-

$\boxed{1}$ $$\frac{2x^2+5}{(x^2-4)(x+3)} = \frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{x+3}$$

$(x-2)(x+2)(x+3)$

$\boxed{2}$ $$\frac{2x^2+5}{(x-4)^3(x+3)} = \frac{A}{x+3} + \frac{B}{x-4} + \frac{C}{(x-4)^2} + \frac{D}{(x-4)^3}$$

$(x-4)(x-4)(x-4)(x+3)$.

نخلص من المختلف بعدين نلحش بالمتكرر

Ex:-

$$\int \frac{1}{x^3+x^2} \, dx$$

$x^3 + x^2 = x^2(x+1)$

$x \cdot x \; (x+1)$

$$\frac{1}{x^3+x^2} = \frac{A}{x+1} + \frac{B}{x} + \frac{C}{x^2}$$

مثل $\frac{1}{2} + \frac{1}{4}$

$$= \frac{A}{x+1} + \frac{Bx+c}{x^2}$$

نوحد المتشابه بعدين نوحد اللي عليه مع غير المتشابه

$$\frac{1}{x^3+x^2} = \frac{Ax^2 + (Bx+c)(x+1)}{(x+1)(x^2)}$$

$$\Rightarrow \quad 1 = Ax^2 + (Bx+c) + (x+1)$$

Let $x = -1 \rightarrow 1 = A$

Let $x = 0 \rightarrow 1 = c$

Let $x = 1 \rightarrow 1 = 1 + (B+1)2$

$$B = -1$$

$$\int \frac{1}{x+1} + \frac{-1}{x} + \frac{1}{x^2} \cdot dx$$

$$= \ln|x+1| - \ln|x| + \frac{x^{-1}}{-1} + c$$

$\boxed{3}$ If $Q(x)$ has Quadratic factor with المميز discriminat < 0 (لا يحلل)

then $\dfrac{P(x)}{Q(x)} = \ldots\ldots + \underbrace{\dfrac{Ax+B}{}}_{\text{اقتران تربيعي}}$

لا يحلل مميزه سالب.

Ex: Write the partial decomposition for the following

$\boxed{1}$ $\dfrac{2x+1}{(x-1)(x^2+1)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+1}$

$\boxed{2}$ $\dfrac{2x+1}{(x-1)(x^2+1)^2} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+1} + \dfrac{Dx+E}{(x^2+1)^2}$

Tues. 4/3/2014

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Ex :-

$$\int \frac{2x^2 - x + 4}{x^3 + 4x} \, dx$$

$$x^3 + 4x = x(x^2+4)$$

$$\frac{2x^2 - x + 4}{x^3 + 4x} = \frac{A}{x} + \frac{Bx + C}{x^2+4}$$

$$\frac{2x^2 - x + 4}{x^3+4x} = \frac{A(x^2+4) + x(Bx+C)}{x(x^2+4)}$$

$$2x^2 - x + 4 = A(x^2+4) + x(Bx+C)$$

let $x = 0 \longrightarrow 4 = 4A \longrightarrow A = 1$

let $x = 1 \longrightarrow B + C = 0$ ---- ①

let $x = -1 \longrightarrow B - C = 2$ --- ②

$2B = 2 \longrightarrow B = 1$

$C = -1$

$$\int \frac{1}{x} + \frac{x-1}{x^2+4} \, dx$$

$$= \int \frac{1}{x} + \frac{x}{x^2+4} - \frac{1}{x^2+4} \, dx$$

$$= \ln|x| + \frac{\ln|x^2+4|}{2} - \frac{\tan^{-1}\left(\frac{x}{2}\right)}{2} + C$$

→ If degree P ≥ degree Q divide then continue.

Ex: $\int \frac{x^4}{x^2-1} \, dx = \int$

$$\begin{array}{r} x^2+1 \\ x^2-1 \overline{) \; x^4 \qquad} \\ \ominus \, x^4 \oplus - x^2 \\ \hline x^2 \\ x^2 - 1 \\ \hline 1 \end{array}$$

$= \frac{x^3}{3} + x + \int \frac{A}{x-1} + \frac{B}{x+1}$

$= \frac{x^3}{3} + x + A.\ln|x-1| + B.\ln|x+1| + C$

Find A & B "partial fraction"

→ **Rationalizing**:

نلغي الجذر ونصير البسط والمقام عبارة عن poly.

① $\int \frac{\sqrt{x+4}}{x} \, dx$

let $x+4 = z^2$

$x = z^2 - 4$

$1\,dx = 2z \, dz$

$\sqrt{x+4} = z$

لو السؤال evaluate نكمله كله
أما Rationalize نوقف لـ أول ما يتحول لـ $\frac{\text{Poly}}{\text{Poly}}$

بس

$\int \frac{z}{z^2-4} \cdot 2z \, dz = 2\int \frac{z^2}{z^2-4} \, dz$

قسمة
مطولة

(2) $$\int \frac{dx}{1+\sqrt[3]{x}}$$

$$x = z^3$$
$$\sqrt[3]{x} = z$$
$$dx = 3z^2 . dz$$

$$3\int \frac{z^2 . dz}{1+z} = 3\int z-1 + \frac{1}{z+1}$$

$$\begin{array}{r|l} & z-1 \\ \hline z+1 & z^2 \\ & z^2+z \\ \hline & -z \\ & -z-1 \\ \hline & 1 \end{array}$$

$$= 3\left(\frac{z^2}{2} - z + \ln|z+1|\right)$$

$$z = \sqrt[3]{x}$$

(3) $$\int \frac{1}{\sqrt{x} - \sqrt[3]{x}} . dx$$

قوة 6 بتخلصني من الجذرين

$$x = z^6 \Rightarrow \boxed{dx = 6z^5 . dz}$$

$$\boxed{\sqrt{x} = z^3} \quad , \quad \boxed{\sqrt[3]{x} = z^2}$$

$$\int \frac{6z^5}{z^3 - z^2} . dz = 6\int \frac{z^5}{z^2(z-1)} . dx$$

$$= 6\int \frac{z^3}{z-1} . dx$$

$$= 6\int z^2 + z + 1 + \frac{1}{z-1}$$

$$\begin{array}{r|l} & z^2+z+1 \\ \hline z-1 & z^3 \\ & z^3 - z^2 \\ \hline & z^2 \\ & z^2 - z \\ \hline & z \\ & z-1 \\ \hline & 1 \end{array}$$

44-

$$6\left(\frac{z^3}{3}+\frac{z^2}{2}+z+\ln|z-1|\right)$$

$$z=\sqrt[6]{x}$$

Ex:-

$$\int \frac{1}{1-\sin x}\, dx$$

مراجعة Calculus (1) limits 4.4

$$\frac{1}{1-\sin x} * \frac{(1+\sin x)}{(1+\sin x)} = \frac{1+\sin x}{1+\sin^2 x} = \int \frac{1+\sin x}{\cos^2 x}\, dx$$

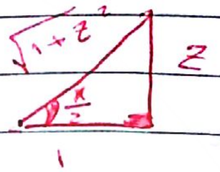
مرافق

$$= \int \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\, dx$$

$$= \int \sec^2 x + \tan x \cdot \sec x \, dx = \tan x + \sec x + C.$$

* Methods of half angle substitution:-

$$\boxed{\text{let } z = \tan\left(\frac{x}{2}\right)}$$



$$\sin 2x = 2\sin x \cdot \cos x$$

$$\Rightarrow \boxed{\sin x} = 2\sin\left(\frac{x}{2}\right) \cdot \cos\left(\frac{x}{2}\right)$$

$$= 2 \cdot \frac{z}{\sqrt{1+z^2}} \cdot \frac{1}{\sqrt{1+z^2}} = \boxed{\frac{2z}{1+z^2}}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\Rightarrow \cos x = \left(\cos\left(\frac{x}{2}\right)\right)^2 - \left(\sin\left(\frac{x}{2}\right)\right)^2$$

$$= \left(\frac{1}{\sqrt{1+z^2}}\right)^2 - \left(\frac{z}{\sqrt{1+z^2}}\right)^2$$

$$= \frac{1}{1+z^2} - \frac{z^2}{1+z^2}$$

$$\boxed{\cos x = \frac{1-z^2}{1+z^2}}$$

$Z = \tan\left(\frac{x}{2}\right) \Rightarrow \tan^{-1} Z = \frac{x}{2} \Rightarrow x = 2\tan^{-1} Z \Rightarrow$

$1\,dx = 2 \cdot \frac{1}{1+z^2} \cdot dz$

$\Rightarrow \boxed{1\,dx = \frac{2}{1+z^2} \cdot dz}$

Ex:- $\int \frac{1}{3\sin x - 4\cos x}\,dx$

Sol:

let $z = \tan\left(\frac{x}{2}\right)$

$\sin x = \frac{2z}{1+z^2}$

$\cos x = \frac{1-z^2}{1+z^2}$

$dx = \frac{2}{1+z^2} \cdot dz$

$3\sin x - 4\cos x = 3\left(\frac{2z}{\sqrt{1+z^2}}\right) - 4\left(\frac{1-z^2}{1+z^2}\right)$

$= \frac{6z - 4 + 4z^2}{1+z^2}$

$$\frac{1}{3\sin x - 4\cos x} = \frac{1+z^2}{4z^2+6z-4}$$

$$\int \frac{1}{3\sin x - 4\cos x} \cdot dx = \int \frac{1+\cancel{z^2}}{4z^2+6z-4} \cdot \frac{2}{1+\cancel{z^2}} \cdot dz$$

$$= \int \frac{1}{2z^2+3z-2} \cdot dz$$

$2z^2+3z-2$

$(2z-1)\,(z+2)$

By partial fraction

$$\frac{A}{2z-1} + \frac{B}{z+2} + \text{---------}$$

Ex:

$\boxed{1}$ $\int x \cdot \sqrt{1-x^4} \cdot dx$

$$= \int x \sqrt{1-(x^2)^2} \cdot dx$$

let $x^2 = z \longrightarrow 2x \cdot dx = dz$

$$\frac{1}{2}\int \sqrt{1-z^2} \cdot dz$$

Trig. Sub. let $z = \sin\theta$ ....

(2) $\int \frac{dx}{\sqrt{e^x+1}}$

$e^x + 1 = z^2$

$\sqrt{e^x+1} = z$

$e^x . dx = 2z . dz$

$dx = \frac{2z}{z^2-1} . dz$

$\int \frac{2\cancel{z}}{z^2-1} . \frac{1}{\cancel{z}} . dz$

$2 \int \frac{1}{\sqrt{z^2-1}} . dz$

$\frac{A}{z-1} + \frac{B}{z+1}$ [Partial].

$\boxed{3}$ $\int x . \tan^2 x . dx$

$= \int x \, (\sec^2 - 1) \, dx$

$= \int \underbrace{x}_{u} \underbrace{\sec^2 x . dx}_{dv} - \int x . dx$

$- \frac{x^2}{2} + c$

(By parts)

④ $$\int \frac{1}{x^4 - x^2} \cdot dx$$

$$x^4 - x^2 = x^2(x^2-1) = x \cdot x\,(x-1)\,(x+1)$$

$$\frac{1}{x^4 - x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{x+1}$$

* imp ⑤ $$\int \frac{e^x}{e^x - e^{-x} + 1} \cdot dx$$

$$\frac{e^x}{e^x - e^{-x} + 1} \cdot \left(\frac{e^x}{e^x}\right) = \frac{e^x \cdot e^x}{e^{2x} + e^x - 1} \cdot dx$$

let $e^x = z$

$e^x \cdot dx = dz$

$$\int \frac{z \cdot dz}{z^2 + z - 1}$$

$z^2 + z - 1$

$= z^2 + z + \frac{1}{4} - \frac{1}{4} - 1$

$= \left(z + \frac{1}{2}\right)^2 - \frac{5}{4}$

$$= \int \frac{z \cdot dz}{\left(z + \frac{1}{2}\right)^2 - \frac{5}{4}}$$

Trig. sub.

let $z + \frac{1}{2} = \frac{\sqrt{5}}{2} \sec\theta$

50

(6) $$\int \frac{\tan x}{\sec^2 x}\, dx$$

$$= \int \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos^2 x}} \cdot dx = \int \frac{\sin x}{\cancel{\cos x}} \cdot \cos^{\cancel{2}} x \cdot dx$$

$$= \int \sin x \cdot \cos x \cdot dx$$

(7) $$\int \frac{\tan^3 x}{\cos^3 x} \cdot dx$$

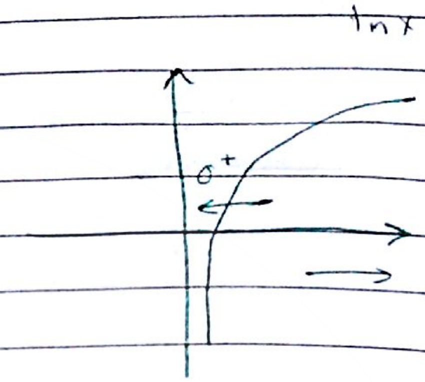
$$= \int \tan^3 x \cdot \sec^2 x \cdot dx$$

$$= \int \tan^2 x \cdot \sec^2 x \cdot (\sec x \cdot \tan x \cdot dx)$$

Scanned by CamScanner

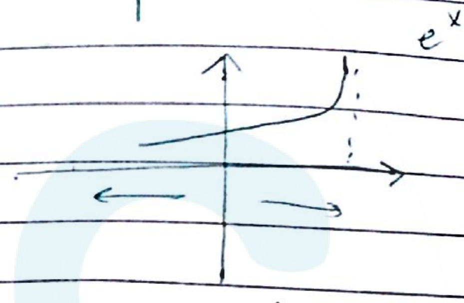
11/3/2014

* Revision :



① $\lim_{x \to \infty} \ln x = \infty$

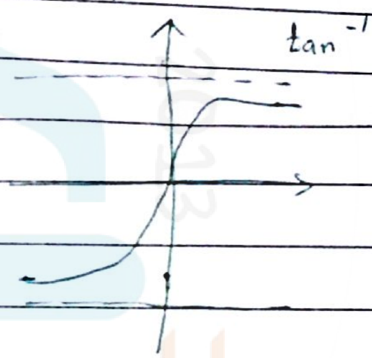
② $\lim_{x \to 0^+} \ln x = -\infty$



③ $\lim_{x \to \infty} e^x = \infty$

④ $\lim_{x \to -\infty} e^x = 0$

⑤ $\tan^{-1} 0 = 0$



⑥ $\lim_{x \to \infty} \tan^{-1} x = \tan^{-1} \infty = \frac{\pi}{2}$

⑦ $\lim_{x \to -\infty} \tan^{-1} x = \tan^{-1} -\infty = \frac{-\pi}{2}$

• Rules

$$\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^x = e^a$$

Ex:-

$$\lim_{x \to \infty} \left(1 - \frac{2}{x}\right)^x$$

$$= \lim_{x \to \infty} \left(1 + \frac{-2}{x}\right)^x = e^{-2}$$

Ex:

$$\lim_{x \to 0^+} x \cdot \ln x \qquad 0 \cdot -\infty \;!!$$

$$\lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x}} \qquad \frac{-\infty}{\infty} \;! \qquad L \text{ Rule}$$

$$\lim \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim \frac{1}{x} \cdot \frac{-x^2}{1}$$

$$= \lim_{x \to 0} -x = 0$$

Ex:- $$\lim_{x \to \infty} x \cdot e^{-x} \qquad \infty \cdot 0 \;!$$

$$\lim_{x \to \infty} \frac{x}{e^x} \Rightarrow \frac{\infty}{\infty} \;!! \quad L' \text{ Rule} \quad \lim_{x \to \infty} \frac{1}{e^x} = \frac{1}{\infty} = 0$$

مهمة

** 7.8 Improper Integrals :-

[1] Infinite intervals

$$\int_a^{\infty} f(x)\,.dx \;,\; \int_{-\infty}^{a} f(x)\,.dx \;,\; \int_{-\infty}^{\infty} f(x)\,.dx$$

الطرف أو الطرفين فيه ∞ أو -∞

53-

$\boxed{2}$. Infinite discontinuity.

ex:- $\int_{0}^{1} \frac{1}{x} \cdot dx$, $\int_{-1}^{1} \frac{1}{x} \cdot dx$, $\int_{0}^{2} \frac{1}{x-2} \cdot dx$

$0 \in [-1, 1]$

ما بيجي كثير

$\boxed{3}$ Infinite interval and infinite discontinuity

دمج الحالة ١ مع ٢

ex:- $\int_{0}^{\infty} \frac{1}{x} \cdot dx$

نكامل بالأول بعدين باخذ ال limit

(1) $\int_{a}^{\infty} f(x) \cdot dx = \lim_{L \to \infty} \int_{a}^{L} f(x) \cdot dx$

if limit exist $(\neq \pm\infty) \Rightarrow$ the integral is

متقارب convergent

if limit $= \pm \infty$, then the integral is divergent

(2) $\int_{-\infty}^{a} f(x) \cdot dx = \lim_{L \to -\infty} \int_{L}^{a} f(x) \cdot dx$ -----

(3) $\int_{-\infty}^{\infty} f(x) \cdot dx = \int_{-\infty}^{c} f(x) \cdot dx + \int_{c}^{\infty} f(x) \cdot dx$

Five Apple

تابع 3

$$= \lim_{L \to -\infty} \int_{L}^{c} f(x).dx + \lim_{M \to \infty} \int_{c}^{M} f(x).dx$$

* At this case if both limits exist, then the integral is convergent, if one of the limits doesn't exit $(\pm\infty)$ then the integral is divergent

conv. + conv. = conv.
conv. + div. $\to$ div.

Ex:-

Evaluate (Is the following cov. or div.) ?

$\boxed{1}$ $\int_{1}^{\infty} \frac{1}{x} . dx$

$$= \lim_{L \to \infty} \int_{1}^{L} \frac{1}{x} . dx = \lim_{L \to \infty} \ln|x| \Big]_{1}^{L}$$

$$\lim_{L \to \infty} (\ln L - \ln 1) = \ln \infty - \underbrace{\ln 1}_{0} = \infty$$

" div. "

$\boxed{2}$ $$\int_1^\infty \frac{1}{x^2} \cdot dx$$

$$= \lim_{L \to \infty} \int_1^L x^{-2} \cdot dx$$

$$= \lim_{L \to \infty} \left. \frac{x^{-1}}{-1} \right]_1^L$$

$$= \lim_{L \to \infty} \left(\frac{-1}{L} + 1 \right)$$

$$= \cancel{\frac{-1}{\infty}}_{0} + 1 = 1 \qquad \text{" Conv. "}$$

$\boxed{3}$ $$\int_1^\infty \frac{1}{x^{\frac{1}{2}}} \cdot dx$$

$$= \lim_{L \to \infty} \int_1^L x^{-\frac{1}{2}} \cdot dx$$

$$= \lim_{L \to \infty} \left. \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right]_1^L$$

$$= \lim_{L \to \infty} \left(2\sqrt{L} - 2\sqrt{1} \right)$$

$$= \infty - 2 = \infty$$

" div "

* Rules

$$\int_1^{\infty} \frac{1}{x^p} \cdot dx = \begin{cases} \text{conv. if } p > 1 \\ \text{with value } = \frac{1}{p-1} \\ \\ \text{div. if } p \le 1 \end{cases}$$

Ex:

$$\int_1^{\infty} \frac{1}{x^{\frac{3}{2}}} \cdot dx$$

$$p = \frac{3}{2} > 1 \quad \text{conv.}$$

$$= \frac{1}{p-1} = \frac{1}{\frac{3}{2}-1} = 2$$

Ex:

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2} \cdot dx$$

$$= \int_{-\infty}^{0} \frac{1}{1+x^2} \cdot dx + \int_{0}^{\infty} \frac{1}{1+x^2} \cdot dx$$

$$= \lim_{L \to -\infty} \int_{L}^{0} \frac{1}{1+x^2} \cdot dx + \lim_{M \to \infty} \int_{0}^{M} \frac{1}{1+x^2} \cdot dx$$

$$= \tan^{-1} x \Big]_L^0 + \tan^{-1} x \Big]_0^M$$

$$= \lim_{L \to -\infty} \left(\tan^{-1} 0 - \tan^{-1} L\right) + \lim_{M \to \infty} \left(\tan^{-1} M - \tan^{-1} 0\right)$$

$$= 0 - \tan^{-1}(-\infty) + \tan^{-1}\infty - \underbrace{\tan^{-1} 0}_{\to 0}$$

$$= -\left(\frac{-\pi}{2}\right) + \frac{\pi}{2} \Rightarrow \frac{\pi}{2} + \frac{\pi}{2} = \pi \quad \#$$

Ex:-

$$\int_0^{\infty} (1-x)\, e^{-x} \, . dx$$

* Sol:

$$\lim_{x\to\infty} \int_0^{L} \underbrace{(1-x)}_{u} \, \underbrace{e^{-x}}_{dv} \, . dx$$

$$= \left((1-x) \frac{e^{-x}}{-1} \Big]_0^{L} - \int_0^{L} e^{-x} \, . dx \right)$$

$$\lim_{x\to\infty} \left(-(1-x)\, e^{-x} + e^{x} \Big]_0^{\ell} \right)$$

$$= \cancel{-e^{-x}} + x e^{-x} + \cancel{e^{-x}}$$

$$\lim_{x\to\infty} \left(x e^{-x} \Big]_0^{\ell} \right) = \lim_{\ell\to\infty} \ell e^{-\ell} - 0$$

$$\infty . 0$$

$$\lim_{\ell\to\infty} \ell e^{-\ell} = \lim_{\ell\to\infty} \frac{\ell}{e^{\ell}}$$

$$0$$

$\equiv$ 0 exist

$\therefore$ conv.

$\boxed{2}$ → if $f(x)$ is cont. on $[a,b)$ a b

$$\Rightarrow \int_a^b f(x)\,.dx = \lim_{L\to b^-} \int_a^L f(x)\,.dx$$

a^+ → b^-

→ if $f(x)$ is cont. on $(a,b]$

$$\Rightarrow \int_a^b f(x)\,.\,dx = \lim_{L\to a^+} \int_L^b f(x)\,.\,dx$$

→ if $f(x)$ is cont. on $[a,b]$ except $c \in (a,b)$

$$\Rightarrow \int_a^b f(x)\,.\,dx = \int_a^c f(x)\,.dx = \int_c^b f(x)\,.dx$$

$$= \lim_{L\to c^-} \int_a^L f(x)\,.\,dx + \lim_{M\to c^+} \int_M^b f(x)\,.dx$$

a c b

c^- → c ← c^+

Ex.: $\int_0^1 \frac{dx}{\sqrt{1-x}}$

Sol.: $\lim_{L \to 1^-} \int_0^L (1-x)^{-\frac{1}{2}} \cdot dx$

$\left. \frac{(1-x)}{} \right]_{-L}^{L}$

$= -2 \sqrt{1-x} \Big]_0^L$

$\lim_{L \to 1^+} \left(-2\sqrt{1-L} + 2\sqrt{1-0} \right)$

$= \cancel{-2\sqrt{0}} + 2$

$= 2$

Conv.

Ex: $\int_1^2 \frac{dx}{1-x}$

$= \lim_{L \to 1^+} \int_L^2 \frac{dx}{1-x}$

$= \frac{\ln|1-x|}{-1} = \lim_{L \to 1^+} \left(-\ln\underbrace{|1-2|}_{0} + \ln\underbrace{|1-L|}_{\ln 0^+}\right)$

$= -\infty$

(div.)

Ex: $\int_0^1 \ln x \, .dx$

$= \lim_{L \to 0^+} \int_L^1 \underbrace{\ln x}_{u} \, . \underbrace{dx}_{dv}$

$= x . \ln x - \int x . \frac{1}{x} . dx$

$= x \ln x - x \Big]_L^1$

$\lim_{L \to 0^+} \left((\ln 1 - 1) - (L \ln L - L) \right)$

$= -1 - \underbrace{0 . \ln 0^+}_{0 . -\infty \to 0} - 0 = -1$

conv

$\lim_{L \to 0^+} L . \ln L$

$= \lim \frac{\ln L}{\frac{1}{L}}$

$\downarrow$

0

فهم علاقة واحد اكبر من التاني ودايماً اكبر

* Comparison Text : (Thm)

Let $f(x)$ and $g(x)$ two cont. funs (متصلة على a, b)

on $[a,b]$ s.t $f(x) \geq g(x)$

① if $\int_a^b g(x)\,.dx$ div. $\Rightarrow$ $\int_a^b f(x)\,.dx$ div.

(الصغيرة) (الكبيرة)

② if $\int_a^b f(x)\,.dx$ conv. $\Rightarrow$ $\int_a^b g(x)\,.dx$ conv

(الكبيرة) (الصغيرة)

*** Ex:** Is the following conv or div.

$$\int_1^\infty \frac{x}{1+x^3}\,.dx$$

Sol:

$$1 + x^3 > x^3$$

$$\frac{1}{1+x^3} < \frac{1}{x^3}$$

ضربت بـ x

$$\frac{x}{1+x^3} < \frac{1}{x}$$

$\int_1^\infty \frac{1}{x^2}\,.dx$ conv. $P = 2 > 1$

$$p = 2 > 1$$

By compasion thm. (conV.)

Ex.:

$$\int_{1}^{\infty} \frac{1+e^{-x}}{x}$$

دائما جواب ال exp.
الأسي
أكبر من صفر

$e^{-x} > 0 \implies 1+e^{-x} > 1$

$$\frac{1+e^{-x}}{x} > \frac{1}{x}$$

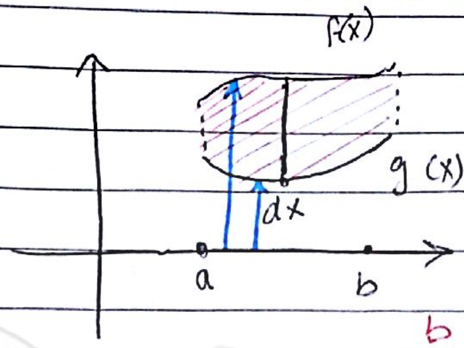
$$\int_{1}^{\infty} \frac{1}{x} \, dx \quad \text{div}$$

$P = 1$
مشهورة

(Div.)   * By comparison.

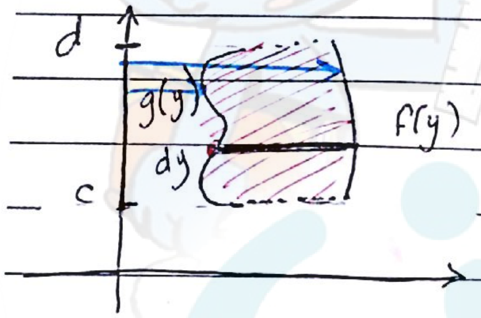
CH 6 :

* Area between Curves :-



$$A = \int_a^b \left(f(x) - g(x)\right) dx$$

عشان أضمن الجواب موجب ← $\boxed{\text{فوق - تحت}}$



$$A = \int_c^d \left(f(y) - g(y)\right) . dy$$

عشان أضمن الجواب موجب ← $\boxed{\text{يمين - شمال}}$

↳ How to change from fn of to fn of y ?

$$y = \sqrt{1 + x} = f(x)$$

$$y^2 = 1 + x$$

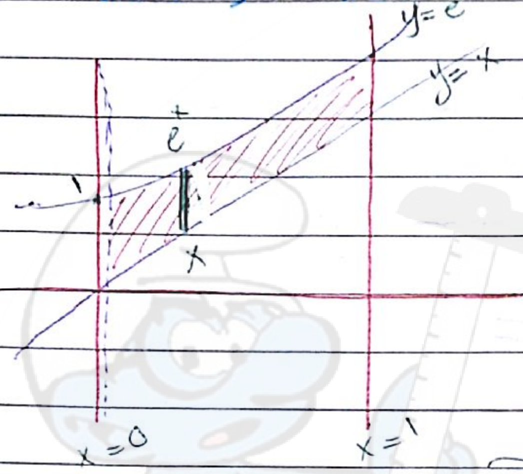
$$y^2 - 1 = x = f(y)$$

e^x , e^{-x} , $|x|$, $\ln x$, x^2 [أتذكر كيف رسمتها * مهم *]

اقتران تربيعي

64 65

Ex: Find the area bounded above by $y = e^x$, bdd below by $y = x$ and bdd From sides by $x = 0$, $x = 1$?



علوي - سفلي

$$A = \int_0^1 (e^x - x) \cdot dx$$

$$= e^x - \frac{x^2}{2} \Big]_0^1$$

$$= e - \frac{3}{2}$$

الأساس إني أطلع الطول من العرض ومن خلاله أطلع المساحة

Ex: Find the area enclosed by $y = x^2$ and $y = 2x - x^2$

Sol: To Find intersection pts

$y = y \Rightarrow x^2 = 2x - x^2 \Rightarrow 2x^2 - x = 0 \Rightarrow$

$2x(x-1) = 0 \quad x = 0 \quad x = 1$

إكمال مربع

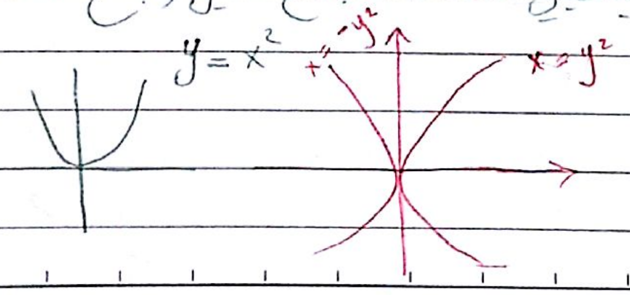
$y = 2x - x^2$
$= -(x^2 - 2x)$
$= -(x^2 - 2x + 1 - 1)$
$= -[(x-1)^2 - 1]$
$-(x-1)^2 + 1$

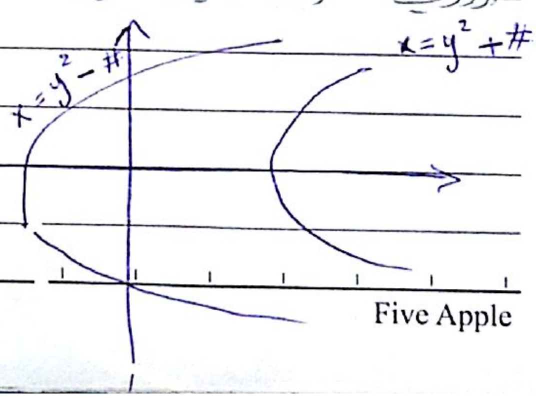
$$A = \int_0^1 (2x - x^2) - (x^2) \cdot dx$$

$$= \frac{1}{3}$$

مراجعة

ضروري أعرف كيف أكوّن معادلة بيعطيني نقاط فبطلع الميل وبطلع معادلة.





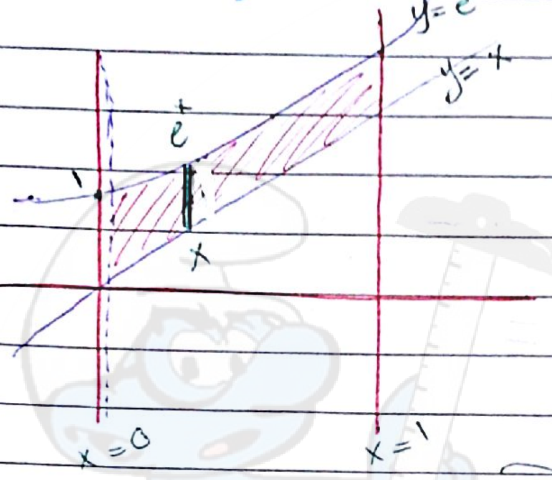
$x = y^2$

" صار عندي اقترانين " $y = \sqrt{x}$ $y = -\sqrt{x}$

e^x, e^{-x}, $|x|$, $\ln x$, x^2 اقتران تربيعي [أتذكر كيف رسمها]

64 65

Ex:- Find the area bounded above by $y = e^x$, bdd below by $y = x$ and bdd From sides by $x = 0$, $x = 1$?



علوي - سفلي

$$A = \int_0^1 (e^x - x)\,.\,dx$$

$$= e^x - \frac{x^2}{2}\Big]_0^1$$

$$= e - \frac{3}{2}$$

الأسهل إني أطلع الطول من العرض ومن خلاله أطلع المساحة

Ex: Find the area enclosed b...

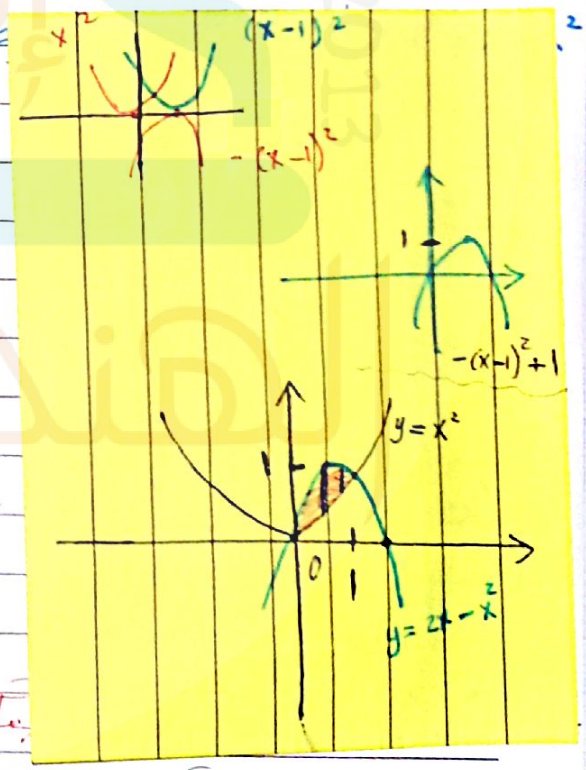
Sol: To Find intersection

$y = y \Rightarrow x^2 = 2x - x^2 \Rightarrow$

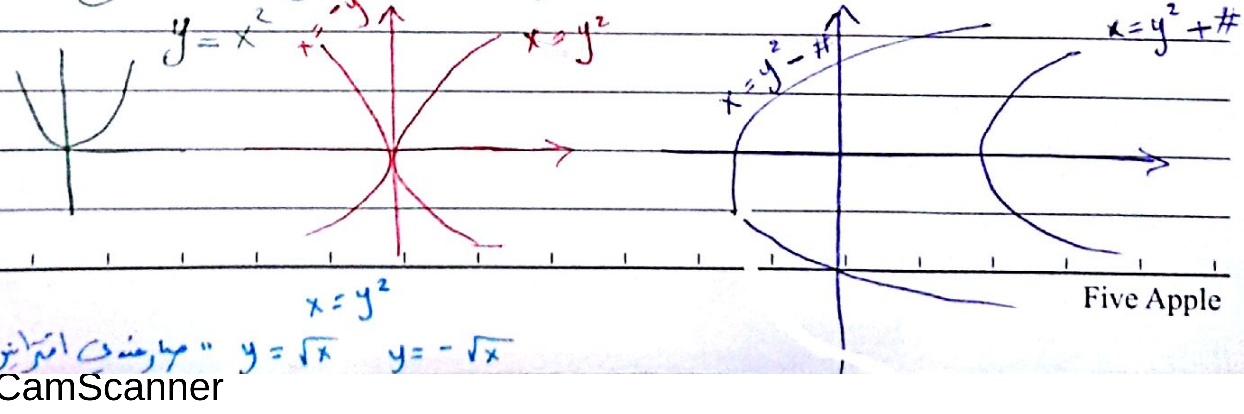
$2x(x-1) = 0 \quad x=0,\ x=1$

$$A = \int_0^1 (2x - x^2) - (x^2)\ .$$

$$= \frac{1}{3}$$



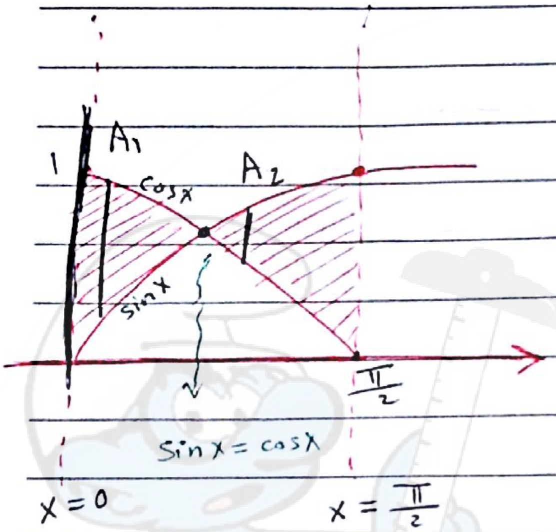
ضروري أعرف كيف أكون معادلة بعطيني نقاط فبطلع الميل وبطلع معادلة



" مهارتك اقترانين " $y=\sqrt{x}$ $y=-\sqrt{x}$

المحصورة

Ex: Find the area enclosed by $y = \sin x$, $y = \cos x$, $x = 0$, $x = \frac{\pi}{2}$?



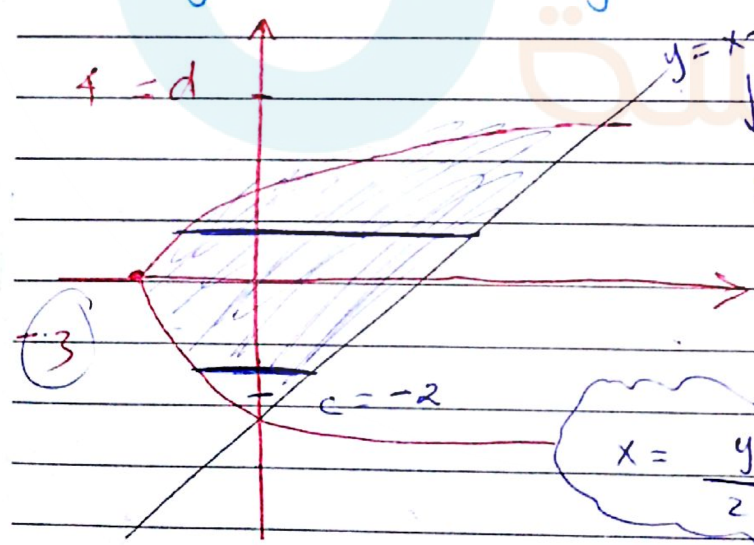
$$\text{Total Area} = A_1 + A_2$$

$$A_1 = \int_0^{\frac{\pi}{4}} (\underset{\text{فوق}}{\cos x} - \underset{\text{تحت}}{\sin x})\, .dx$$

$$A_2 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x)\, .dx$$

ثم نجمعهم

Ex:- Find the area enclosed by the line $y = x - 1$ and $y^2 = 2x + 6$?



$$\boxed{x = y + 1}$$

$$\boxed{x = \frac{y^2}{2} - 3}$$

$$x = \frac{y^2}{2} - 3$$

67-

To Find intersection pts $\Longrightarrow$ $x = x$

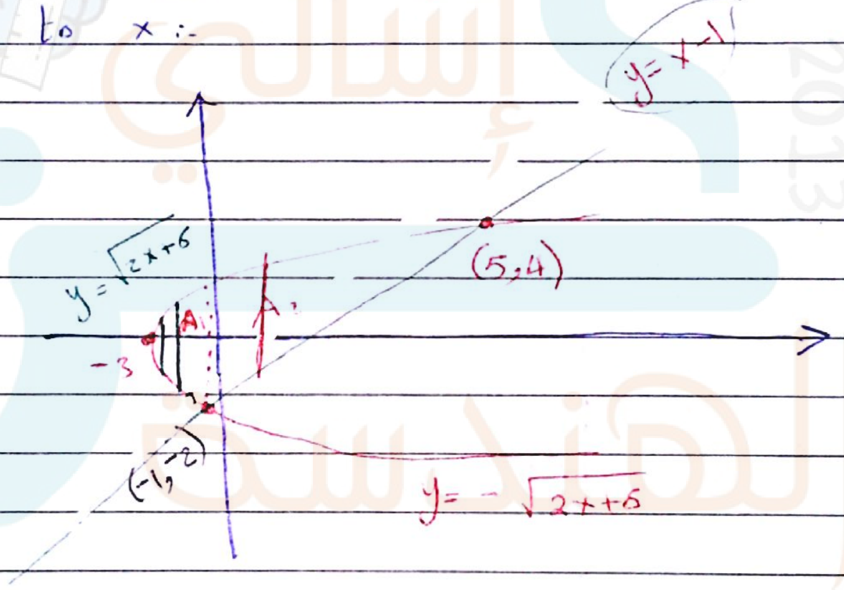
$$\Rightarrow y+1 = \frac{y^2}{2} - 3 \Rightarrow 2y + 2 = y^2 - 6$$

$$y^2 - 2y - 8 = 0 \longrightarrow (y-4)(y+2) = 0$$

$y = 4$ $\quad$ $y = -2$

$$A = \int_{-2}^{4} (y+1) - \left(\frac{y^2}{2} - 3\right) \cdot dy$$

* with respect to x :



من الرسم $\quad (y-4)(y+2) = 0$

$y = 4 \qquad y = -2$

$\downarrow \qquad \downarrow$

$x = y+1 \qquad x = y+1$

$x = 5 \qquad x = -1$

Total Area $= A_1 + A_2$

$\longrightarrow$ تابع

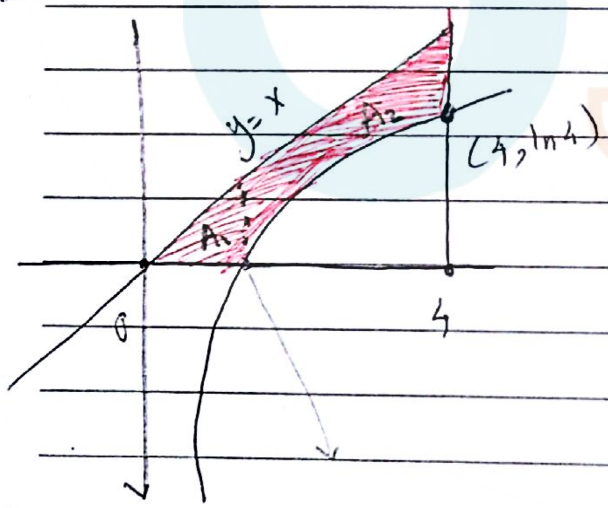
$$A_1 = \int_{-3}^{-1} \sqrt{2x+6} - -\sqrt{2x+6}\;.dx = 2\int_{-3}^{-1} (2x+6)^{\frac{1}{2}}\;.dx$$

$$= \left.\frac{2(2x+6)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)(2)}\right]_{-3}^{-1} \ldots\ldots$$

$$A_2 = \int_{-1}^{5} \sqrt{2x+6} - (x-1)\;.dx$$

$$= \left.\frac{(2x+6)^{\frac{3}{2}}}{\frac{3}{2}(2)} - \frac{x^2}{2} + x\right]_{-1}^{5} = \ldots$$

مهم سؤال امتحان ★★★★ Q1 :- Find the area of the shaded region.



$y = \ln x$ هاي قيمتها 1

$\ln 1 = 0$ لأنه

ح جزئيا

Total Area = $A_1 + A_2$

$$A_1 = \int_{0}^{1} x - 0\;.dx \ldots\ldots$$

$$A_2 = \int_{1}^{4} x - \ln x\;.dx$$

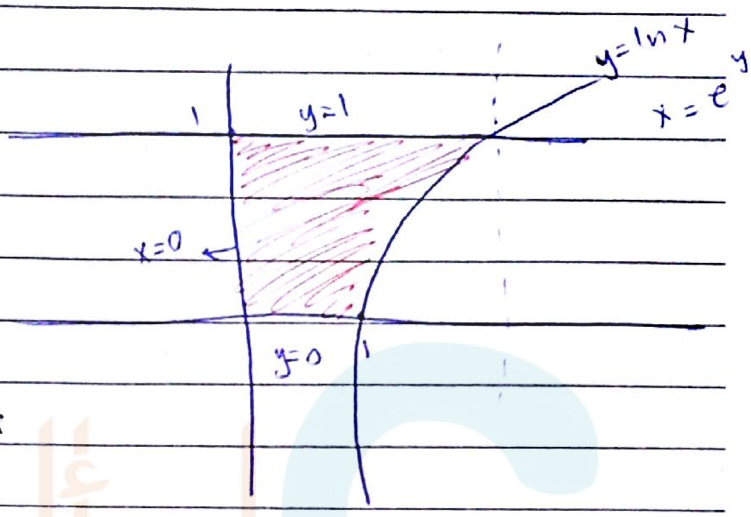
$$A = \frac{x^2}{2} - \int_{1}^{4} \ln x\;.dx$$

By Parts.

أسهل

* تكامل dx لازم كل اشي بدلالة x

* تكامل dy لازم كل اشي بدلالة y



تكامل على dx

$$\int_0^1 (1-0)\,dx + \int_1^e (1-\ln x)\,dx$$

يمين - شمال

$\int \ln x$

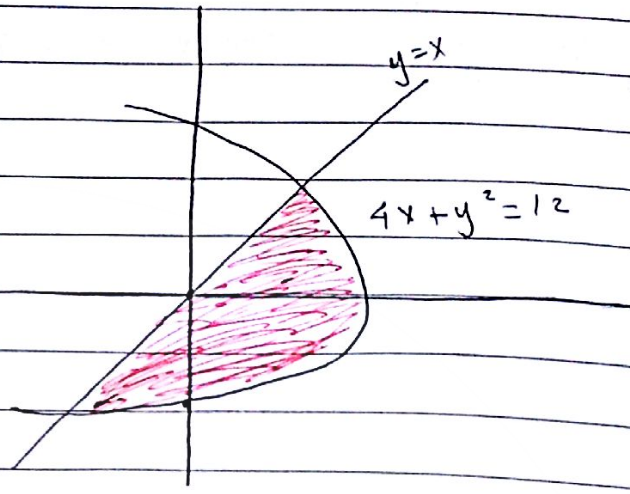
لازم تحل بالأجزاء

تكامل على dy

$$\int_0^1 e^y - 0 \quad dy$$

علوي - سفلي

بدنا نكامل dy يعني كل اشي بالسؤال
لازم بدلالة dy



$$4x + y^2 = 12$$

$$4x = \frac{12 - y^2}{4}$$

$$\int \frac{12 - y^2}{4} - y \; dy$$

$$4y + y^2 = 12$$

$$y^2 + 4y - 12 = 0$$

$$(y+6)(y-2) = 0$$

$$y = -6 \qquad y = 2$$

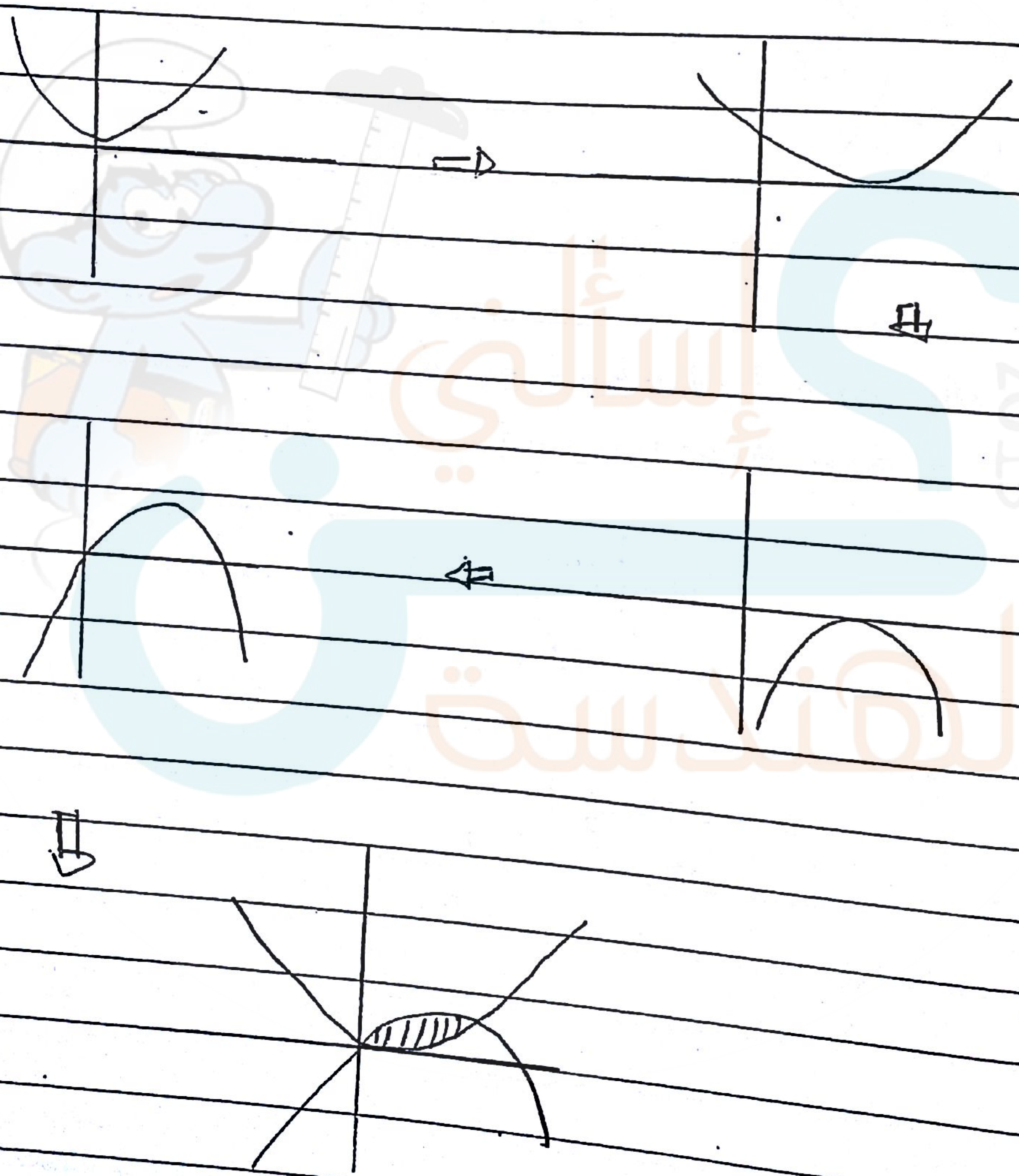
$$\int_{-6}^{2} \frac{12 - y^2}{4} - y \, . dy$$

E.x: Find Area enclosed by $y=x^2$ & $y=2x-x^2$

$y = 2x - x^2 = -(x^2 - 2x)$

$= -(x^2 - 2x + 1 - 1) \Rightarrow -((x-1)^2 - 1)$

$1 - (x-1)^2$



to find intersection points :

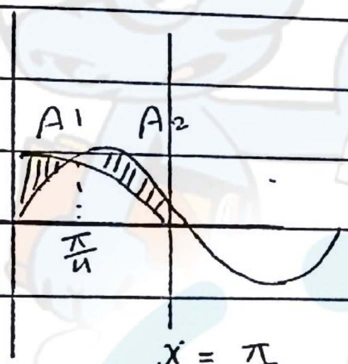
$y = y \longrightarrow x^2 = 2x - x^2$

$2x^2 - 2x = 0$

$2x(x-1) = 0 \longrightarrow \boxed{x=0} \quad \boxed{x=1}$

$A = \int_0^1 (2x - x^2 - x^2) \Rightarrow \int_0^1 (2x - 2x^2) = \left(x^2 - \frac{2}{3}x^3\right)\Big]_0^1$

Ex: Find the area between the curves $y = \sin x$, $y = \cos x$, $x = 0$ & $x = \frac{\pi}{2}$?!

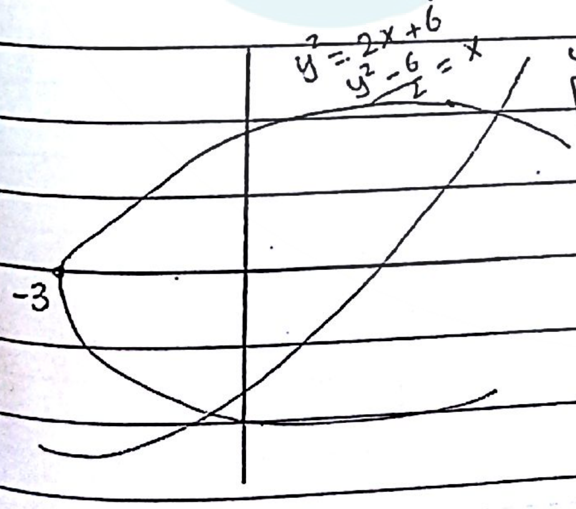


$A_{Total} = A_1 + A_2$

$A_1 = \int_0^{\frac{\pi}{4}} (\cos x - \sin x).dx$

$A_2 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x).dx$

Ex: Find the Area enclosed by the line $y = x - 1$ & $y^2 = 2x + 6$?!



$y = x - 1$

$\boxed{x = y + 1}$

to find intersection point.

$y + 1 = \frac{y^2}{2} - 3$

$y + 4 = \frac{y^2}{2}$

$2y + 8 = y^2$

$y^2 - 2y - 8 = 0$

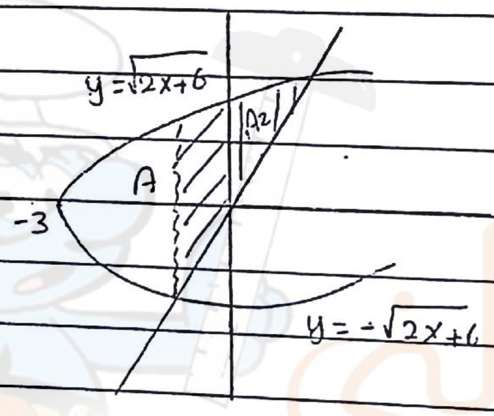
$(y-4)(y+2) = 0$

$\boxed{y = 4} \quad \boxed{y = -2} \quad \Rightarrow$

$$\int_{-2}^{4} (y+1) - \left(\frac{y^2}{2} - 3\right) \cdot dy \implies \int_{-2}^{4} \left(y + 1 - \frac{y^2}{2} + 3\right) dy$$

$$\left.\frac{y^2}{2} + y - \frac{y^3}{6} + 3y\right]_{-2}^{4}$$

To find Area w.r.t x:



$$A = \int_a^b (f(x) - g(x)) \cdot dx$$

To find intersection point:

$x = x \rightarrow y + 1 = \frac{y^2}{2} - 6$

$\boxed{y = 4}$ $\boxed{y = -2}$

$\boxed{x = 5}$ $\boxed{x = -1}$

$\frac{y^2 - 6}{2} = x \implies y^2 - 6 = 2x$

$y = \pm\sqrt{6 + 2x}$

$A_{total} = A_1 + A_2$

$A_1 \int_{-3}^{-1} \left(\sqrt{2x+6} - (-\sqrt{2x+6})\right) \cdot dx$

$= \int_{-3}^{-1} (2\sqrt{2x+6}) \cdot dx = \int_{-3}^{-1} 2(2x+6)^{1/2} \cdot dx$

$= \left.\frac{2(2x+6)^{3/2}}{\frac{3}{2}(2)}\right]_{-3}^{-1}$

$A_2 = \int_{-1}^{5} \sqrt{2x+6} - (x-1) \cdot dx$

$\left.\frac{(2x+6)^{3/2}}{(3/2)(2)} - \frac{x^2}{2} + \frac{x^2}{2} + x\right]_{-1}^{5}$